Errata/Corrigenda (as of 11/21/16) for

STOCHASTIC MODELS, INFORMATION THEORY, AND LIE GROUPS, Vol 1.
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The (mostly typographical) errors listed below apply to the first (Sept 2009) printing of Vol 1., which includes more than 3000 equations and 168 exercises.

p.2-3, For (1.4)-(1.5) to match Fig 1.1., the reference frame attached to the robot should have its x-axis pointing in direction of forward motion instead of y axis.

p. 16, Sentence with footnote 11 should end with α rather than i; Last line on page, "of arbitrary" should be "of an arbitrary"

p. 17, 3rd line from bottom, $g(f(y))$ should be $g(f(x))$.

p. 29, in (1.56), either the $<$ signs should all be $\leq$, or parenthesis starting to left of α_k and ending at the far right should be added.

p. 33, 3rd unlabeled equation missing dx.

pp. 33-34, In Equations 2.4 and 2.5 the minus signs should be pluses.

p. 41, 7th line, "variance" should be "covariance".

p. 42, In (2.29), Σ_2 should be Σ_{22}. Also, while not stated, the matrices M, G, A in Section 2.2.2 are assumed to be symmetric.

pp. 43-47, The identity matrix I should be denoted as $\mathbb{I}$ for consistency.

pp. 45-51, In Sections 2.4-2.6, all instances of $\rho(x, \mu, \sigma)$ should be $\rho(x, \mu, \sigma^2)$.

p. 47 $\mathbf{x}^T\Sigma_0^{-1}\mathbf{x} \longrightarrow \frac{1}{2}\mathbf{x}^T\Sigma_0^{-1}\mathbf{x}$ In the long eqn in middle of page, $\phi 0$ should be $\phi = 0$ at lower bound of middle integral, there should be a factor of $\sin\theta$ in the volume element and the result of rightmost equality should contain $r(\Sigma_0, a)$ in the denominator.

p. 51, In (2.56) ∂f^2 should be $\partial^2 f$.

p. 55, line 8, f missing at end of line.

p. 57, line 8, ∂f^2 should be $\partial^2 f$.

p. 60, In (2.84) ∂f^2 should be $\partial^2 f$.

p. 66, In Eq. 3.7, $f(\mathbf{y})$ should be $\rho(\mathbf{y})$ for consistency.

p. 69, line 7, the statement assumes that $f(x_1, x_2) = f_1(x_1)f(x_2)$.

p. 69, line above Eq. 3.15, dx_2dx_2 should be dx_1dx_2.

p. 71, In last paragraph, "condition expectation" should be "conditional expectation." And (3.24) has one too many parenthesis.

p. 73, In (3.25), $\langle\rho_i'/\rho_i\rangle^2$ should be $\langle(\rho_i'/\rho_i)^2\rangle$.

p.75 To be consistent with the literature the definition of $N(f)$ in (3.33) should be divided by $2\pi e$. This does not affect the inequality.

p. 77, in equation mid page l.h.s $\nabla^T_{\boldsymbol{\phi}}\tilde{f}(\boldsymbol{\phi})$ should be $\nabla^T_{\boldsymbol{\phi}}f(\mathbf{x}(\boldsymbol{\phi}))$. This means that in the equations that follow in Section 3.3.4 $J(\boldsymbol{\phi})/|J(\boldsymbol{\phi})|$ should be $J(\boldsymbol{\phi})$ and the final equation in that section should be $J(\boldsymbol{\phi})J^T(\boldsymbol{\phi}) = \mathbb{I}$, indicating that the Fisher information divergence is only invariant when the Jacobian is orthogonal. And so in the 1D case it is only invariant under translations. See addendum.

p. 82, Second to last line, $m \times m$ should be $p \times m$; In (3.51)-(3.52) $m \longrightarrow n$ and

$$\frac{\partial}{\partial\theta\partial\theta^T} \longrightarrow \frac{\partial^2}{\partial\theta\partial\theta^T}$$

p. 96, In the statement of Ex. 3.2, $\left[\frac{\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2}(\mu_1-\mu_2)^2\right]^2$ should be $\left[\frac{(\mu_1-\mu_2)^2}{\sigma_1^2+\sigma_2^2}\right]$.

p.97, In (3.87) change $\rangle x_1\rangle$ to $\rangle|x_1\rangle$ and in (3.88) change $\rangle z\rangle$ to $\rangle|z\rangle$, and in Ex 3.12, $RR^T = I$ should be $RR^T = \mathbb{I}$

p. 99 ref 44 should read: Smith, S.T., "Covariance, Subspace, and Intrinsic Cramér-Rao Bounds," *IEEE Trans. Signal Processing*, 53(5):1610-1630, May 2005.

p. 106, line 16, "variance" should be "covariance"

p. 107, In the Chapman-Kolmogorov equation, (4.14), $p(\mathbf{x}_1, t_1; \mathbf{x}_3, t_3)$ should be $p(\mathbf{x}_1, t_1 \,|\, \mathbf{x}_3, t_3)$.

p. 108, Since in this context $t_{i-1} > t_i$, it would be better to write (4.15) as $p(\mathbf{x}_{i-1}, t_{i-1} \,|\, \mathbf{x}_i, t_i) = p(\mathbf{x}_{i-1}, t_{i-1} - t_i \,|\, \mathbf{x}_i, 0)$. The unlabeled equation that follows it should then read $p(\mathbf{x}_{i-1} \,|\, \mathbf{x}_i, t) \doteq p(\mathbf{x}_{i-1}, t \,|\, \mathbf{x}_i, 0)$ where $t = t_{i-1} - t_i$. See addendum for details. Also on p. 108, at the end of Section 4.3 is an extraneous letter "i" in the text. Note: (4.17) is the pdf for a unit strength Wiener process, and so in the sentence that follows "... is called a *Wiener process* of strength σ_i^2." should read "... is called a *Wiener process* of unit strength." Wiener processes of arbitrary strength are discussed in Section 4.4.2. In (4.18), $\rho_i(x_i; s_j, t_j)$ should be $\rho_j(x_j; s_j, t_j)$.

p. 109, In unlabeled eqn between (4.22) and (4.23), upper bound in sum should be changed from $1/\Delta t$ to $t/\Delta t$.

p. 114, In (4.41) and the unlabeled equation that follows it, it was assumed (without being stating) that $t_k - t_{k-1}$ was selected as t/n. Therefore, $1/n$ should be t/n. Alternatively (and more generally) $1/n$ could be removed and $(t_k - t_{k-1})$ inserted as a multiplicative weight inside the summations in these two equations.

p. 115, To avoid problems, set $T = 1$ everywhere. Also, at the end of the third paragraph, "$1\sqrt{n}$" should be "$1/\sqrt{n}$".

p. 116, In (4.47) τ should be t.

p. 118, 7th line, $F(T)$ should be $F(\tau)$.

p. 118-119, Let $\mathbf{y} = \mathbf{x}(t + dt)$ and $\mathbf{x} = \mathbf{x}(t)$ and replace $\mathbf{x}|\mathbf{y}$ with $\mathbf{y}|\mathbf{x}$ in the conditionals in unlabeled equations between (4.51) and (4.52).

p. 119, in (4.54) and (4.55), ∂f_i^2 should be $\partial^2 f_i$.

p. 120-1, Though the result in (4.61) is correct, the proof of the Fokker-Planck equation has a subtle error that makes it difficult to go from (4.60) to (4.61). This is fixed by either reversing the roles of $\mathbf{x}$ and $\mathbf{y}$ in each transition probability in the proof, or alternatively by replacing $\epsilon(\mathbf{y})$ by $\epsilon(\mathbf{x})$ and replacing the corresponding integration over $\mathbf{x}$ with integration over $\mathbf{y}$. See addenda for clearer versions of proof. 5th line from bottom of p. 120, $\partial x_j \partial x_k$ should be $\partial x_j \partial x_i$. Also, in 3rd line of p 121, $\partial \xi_k$ should be $\partial \xi_i$.

p. 126, in 7th line $\sum_{jk}$ should be $\sum_{ij}$ in all three places.

p. 133, In the unlabeled equation above (4.97), x_1^{-1} should be x_1^{-2}.

On that same page, in the footnote I should be $\mathbb{I}$.

p. 135, in unlabeled equation at bottom of page $\mathbf{a}(\mathbf{q}, t)$ should be $\mathbf{a}(\mathbf{q}, t)dt$.

p. 136, in unlabeled equation at top of page $\mathbf{a}^s(\mathbf{q}, t)$ should be $\mathbf{a}^s(\mathbf{q}, t)dt$; Exercise 4.2 could have been stated for the multidimensional case with $\boldsymbol{\mu}$ instead of μ and Σ instead of σ^2.

p. 138, In ref 2, Onsager should be capitalized.

p. 145, In (5.4) $1/L_1$ should be removed.

p. 149, Final paragraph, first word "Since" should be "Letting"

p. 151, 2nd line, $\mathbf{k}(t)$ should be $\mathbf{k}_0 + t(\mathbf{k}_{goal} - \mathbf{k}_0)$ (The initial $\mathbf{k}_0$ was missing.). The algorithm in (5.16) can be made more stable by interlacing Jacobian updates of $\mathbf{k}$ as described in Addendum.

p. 161, eqn. below (5.45), $J(\mathbf{v})$ should be $J(\mathbf{s})$.

p. 164, eqn. at bottom of page, q_1' and q_2' should be q_i' and q_j'.

p. 165, In unlabeled eqn at top of page, $\kappa_g(s)$ should be $\kappa_g(s)\mathrm{m}(s)$; In (5.65) $|G|^{-\frac{1}{2}} \longrightarrow |G|^{-1}$

p. 167, 5th line, R^2 missing in front of integral for F.

p. 173, In sentence above (5.89), θ should be α.

p. 187, In Ex. 5.10, $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ should be $|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$ since volume is non-negative; (5.133), in denominator of m, g_{21} should be g_{22}.

p. 188, In Ex 5.19, $x'(t)$ should be $x_1(t) \doteq x(t + 2\pi/3)$, $x''(t)$ should be $x_2(t) \doteq x(t + 4\pi/3)$ and "Rastamian" should be "Rostamian." Also, in Ex. 5.20, δt should be Δt

p. 198, in (6.15), $<$ should be $\leq$; and in (6.17), $+\frac{\partial \tilde{b}_{13}}{\partial x_2}$ should be changed either to $-\frac{\partial \tilde{b}_{13}}{\partial x_2}$ or $+\frac{\partial \tilde{b}_{31}}{\partial x_2}$.

p. 206, 4 lines below (6.32), φ_2 should be $\varphi_2(\mathbf{w})$.

p. 213, penultimate paragraph in Section 6.6, the last sentence "... and a vector in the tangent space ..." should be "... and the tangent space ..."; 4th line from bottom of page, it would be more clear to replace "... vectors interpreted in the form of v ..." with "... vectors $\mathbf{v}(\mathbf{x})$ interpreted in the form of components $v_i(\mathbf{x})$ or fields $\mathcal{V}$ ..."

p. 217, In (6.81) $d\phi$ should be $rd\phi$ in two places; In eqn below (6.82), 2nd and 3rd instances of b_1 should be b_2 and b_3

p. 218, In the computation of $d(\zeta^*\omega_1)$ in Example 4, the error from p. 217 persists. In addition "d[" should be "d[(" in two places.

p. 231, At top of pages, $\Lambda^2(A) \longrightarrow \Lambda^3(A)$. In Exercise 6.17, the right-hand-side should either have a multiplicative factor of $[(n-p)!]^{-1}$ or $\pi \in \Pi_n$ should be replaced with $\pi \in \sigma \in \Pi_{n-p} \backslash \Pi_n$. See addendum to Chapter 6 for details.

p. 232, the roles of x and y should be reversed

p. 235, Four lines above 7.1.2, "are know" $\longrightarrow$ "are known"; 9 lines into Sec. 7.1.2, eqn should be $V(A \cdot C) = |\det A| \cdot V(C)$.

p. 237, Section 7.1.3. For consistency of notation f_C should be denoted I_C.

p. 243 The 11 and 22 entry in b_2' should be swapped.

p. 244, Fig 7.3, lower left 'R' should be flipped around y axis. (This error was introduced when the publisher's artist 'fixed' the author's original figure). See addendum for original figure.

p. 268, 9th line "... $(k+1)$-form whereas ..." should be "... $(k+1)$-form. Whereas ..."

p. 269, At end of Section 7.61 there is an extraneous typesetting command "indexRiemannian!manifold" that should be ignored.

p. 277, 4th line in Section 7.6.3, "Ie" should be "If".

p. 282 In (7.115), Γ_{ij}^{k} are not the Christoffel symbols.

p. 280, Reference to Figure 7.7.1 should be Fig. 7.6.

p. 284, Ex 7.2, the range of values should be $0 \leq \theta, \phi/2 \leq 2\pi$.

p. 285, Second entry in the vector in Ex 7.5 should be $r \sin\theta$ (not $r\cos\theta$).

p. 290, Same subtle problem as in the derivation of the regular Fokker-Planck equation on pp. 120-1 (though the result is correct). Also, in (8.1), $\mathbf{h}(\mathbf{q},t)$ should be $\mathbf{h}(\mathbf{q},t)dt$

p. 293, in (8.11) 0 should be $\mathbb{O}$ and in (8.13), $\mathbf{h}^s(\mathbf{q},t)$ should be $\mathbf{h}^s(\mathbf{q},t)dt$

p. 294, in (8.17) and (8.19), $\mathbf{b}(t)$ should be $\mathbf{b}(t)dt$

p. 298, in (8.33) $\mathbf{e}_i$ should be $\mathbf{e}_1$, and in unlabeled equation just above it $\frac{\partial^2 f}{\partial \theta^2}$ should be replaced with $\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial f}{\partial\theta})$. (8.33) is missing dt

p. 299, Equation 8.36 is missing an R in the first term on the right-hand side of the equality, and the nonzero entry of E_{ij} is $1/\sqrt{2}$ (not 1).

p. 306, Mid page "Taylor" should be "Fourier"

p. 319, Line 7, "(A.10)-(A.13)" should be "(A.10)-(A.12)". Note that the complex version of (A.13) requires $(\mathbf{x},\mathbf{y})$ being replaced with $|(\mathbf{x},\mathbf{y})|$.

p. 324, Line 10, for consistency S_3 should be Π_3.

p. 339, In second unlabeled equation between (A.78) and (A.79), second instance of $a_{ii}x_{i1}$ in determinant should be $a_{ii}x_{i2}$.

p. 341, In eqn at bottom of page the locations of A and B should be switched.

p. 344, six lines from bottom, should be $\alpha = c\,\|\mathbf{x}_0\}\|$ and $v(t) = c\,\|B(t)\|$.

p. 354, 3rd line of A.11.6, $X(t_0)$ missing after $\Phi(t,t_0)$.

p. 357, Exercise A.16, include $|a\phi(x) - b\phi(y)| \leq \phi(ax+by)$.

p. 358, Exercise A.26, "polynomial" should be "equation", $\mathbf{A}$ should be A, and $\mathbf{0}$ should be $\mathbb{O}$.